To prove that Ω is not in the range of any local projector, i.e. it is not an eigenvector of any local observable

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Assume $P_O \Omega = k \Omega$

Then $P_O^2 \Omega = k^2 \Omega = P_O \Omega = k \Omega$

So $k = 0$ or 1

Furthermore $(P_O - kI)\Omega = 0$

So by Reeh-Schlieder

$$P_O - kI = 0$$

or $P_O = kI$, with $k = 0$ or 1

Hence, arguing contrapositively,

$(P_O \neq 0 \text{ or } I) \Rightarrow P_O \Omega \neq k \Omega$

Q.E.D

The Sigma Club

ONE-DAY CONFERENCE ON PHILOSOPHY OF PHYSICS

SATURDAY JUNE 6, 1992

G. Fleming

10.00 - 10.30: Roland Sypel (Oxford): When is a Physical Theory Relativistic?

10.30 - 11.00: Tim Budden (Oxford): The Principle of Relativity and the Isotropy of Boosts

J. B.

11.00: Coffee

[illegible] 11.30 - 12.00: Constantine Pagonis & Rob Clifton (Cambridge): Hardy's Non-locality Theorem for N spin-½ Particles   L. Hardy

M.R 12.00 - 12.30: Harvey Brown (Oxford): Partial Absorption in Neutron Interferometry

Bob Weingard 12.30: Lunch

Redh. 2.00 - 3.00: Mark Hogarth (Cambridge): Cosmic Censorship

Fleming 3.00 - 4.00: Gordon Fleming (Pennsylvania): A Critique of Elements of Reality in GRWP Dynamical Reduction Models

NH

4.00: Tea

H.B 4.30 - 5.30: Michael Redhead (Cambridge): Localization and the Vacuum

J.B. 5.30 - 6.30 R. Weingaard (Rutgers): Some Remarks on Field Theory

HPS Dept, Free School Lane, Cambridge

No Registration Fee: All Welcome

Localization and the Vacuum Content

1. If we are in a localized state there is non-vanishing probability of detecting (nonlocally) any particle state. This is a sort of dual to Malament's theorem (?)

2. Experimentalists regard large ($\gg \hbar/mc$) as infinite. This is true For All Practical Purposes, but is not exactly true (cf Bell).

3. Theoreticians think of particle states as collision states or asymptotic states.

Two approaches: Extension of axiomatic RQFT cf Haag-Ruelle theory or accommodate in local algebra framework – cf Araki etc.

4. Correlations in vacuum fall off exponentially with distance – cf Cluster theorems of Fredenhagen and others.

5. Practical problems of doing "source-free" Bell experiments

as in Summers & Werner & Landau (1985)

6. Interpretation of Reeh-Schlieder theorem in terms of "tweaking" the vacuum — involves selective operations — cf Licht on local states

7. Is what we call a particle just a semantic convention? — "What's in a rose..."
when we stretch concepts we may have to split meanings

Classical particle has localization & definite no. N

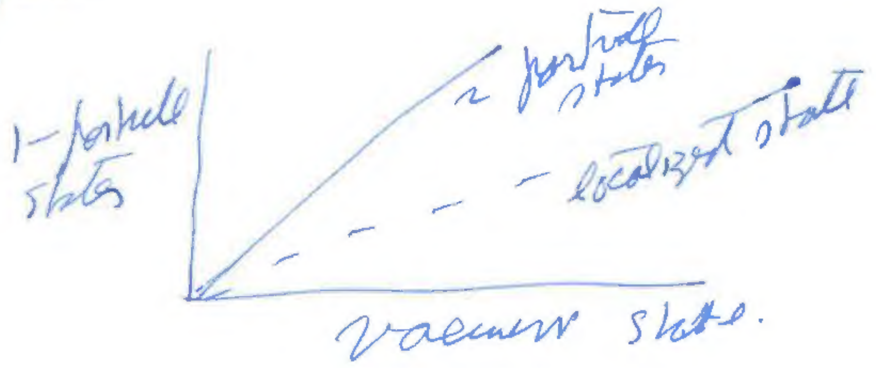
Quantum field
- particle 1: localized but indefinite N
  This is Malament's approach
- particle 2: definite N, but not localized
  This is Redhead's approach

8. All states of a field are just that — states of a field — some of them may be called particle states but this locution can be misleading, lead to apparent paradox so it is best left alone!

Localization and the Vacuum

(1) In RQFT local quantities such as $Q(x)$ do not commute with Number operator N

So eigenstates of $Q(x)$ are inclined to the number axes which scaffold the Hilbert space



Hence near the vacuum state there is necessarily transition probability to any localized state (i.e. can be detected by a localized measurement)

pure particle states are not localized, require detectors everywhere.

N.B. 1 in RQFT $N(x)$, $N(x')$ do not commute. so we cannot measure them simultaneously.

N.B. 2 $N(x)$ in RQFT is not an observable.

In NRQFT, by contrast, $N(x)$ is observable and we measure N by measuring all the $N(x)$ simultaneously.

In order to measure N in RQFT we measure total momentum in the field. Do not attempt localization (to better than $\hbar/mc$, or λ for photons).

Note also we can try to measure $N(\xi)$, ξ is non-x localization but ξ is spread out with regard to x ('ξ' F-W-transform). $N(\xi)$ is not diagonal w.r.t. x. It is a nonlocal operator in x-space.

(2) How to characterize localized state

Proposal 1. $A(O)\Omega$ is local if $A(O)$ is local. — no good: $1 \in R(O)$ so then makes Ω local, etc.

Proposal 3. $A(O)\Omega$ is a localized state if $P_{A(O)\Omega} \in R(O)$.

Theorem. $\mathrm{Prob}(\Omega \to X)$, where X is any localized state $\neq 0$.

Query $H(x)$ commutes with $H(x')$ and hence with total energy H

So ~~eigenstates~~ eigenspace of H is invariant under $H(x)$

But Ω is unique least energy state

So is not Ω an eigenstate of $H(x)$?

Resolution, eigenvalues of H is ∞ ?

So, in the relevant sense, Ω and many-particle states are degenerate w.r.t. total Hamiltonian!

Malament's assumptions

Q. field is $\langle \mathcal{H}, O \mapsto R(O), \underline{a} \to U(\underline{a}), \Omega \rangle$

$R(O)$ is a von Neumann algebra on $\mathcal{H}$.

O is a bounded open set of points in Minkowski space.

$U(\underline{a})$ is a representation of translations in space-time.

Ω is the vacuum.

Theorem!

χ state of field, $p(\chi)$ = probability that detector fires, then if $p(\chi)$ is not independent of χ, $p(\Omega) \neq 0$, provided detector is localized.

Theorem 2

Under same assumptions, detector firing when localized outside O, is always correlated with some operator $A(O) \in R(O)$.

Reeh - Schlieder Theorem

For any O, Ω is cyclic for $\mathcal{H}$, with respect to $R(O)$

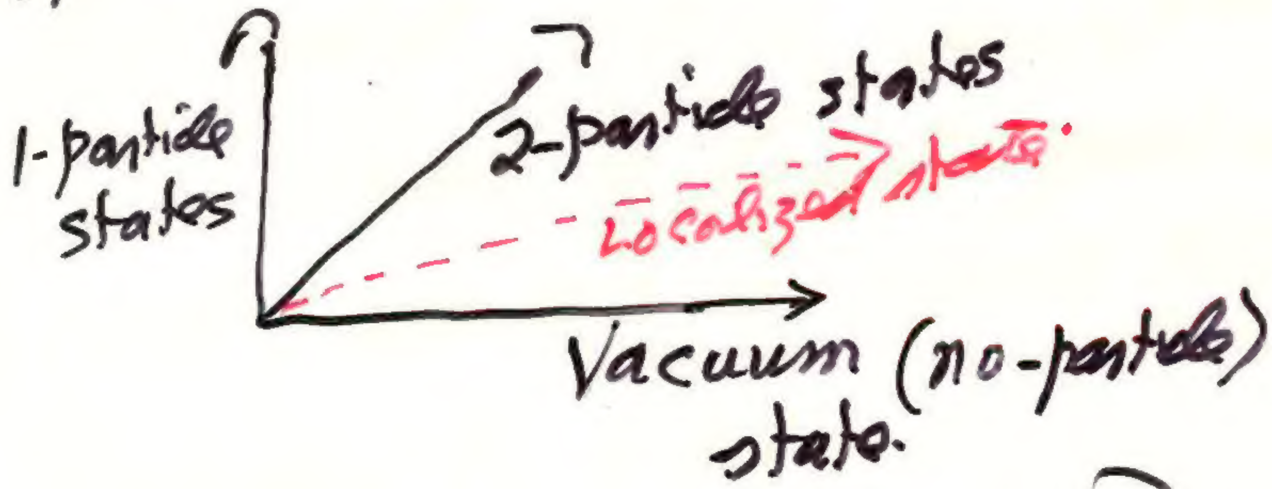
corollary : Ω is a separing vector for any $R(O)$

i.e. $A(O)\Omega = \underline{0}$

$\Longrightarrow A(O) = \underline{0}$

The Relativistic Vacuum

In RQFT localized quantities $Q(x)$, such as charge densities, do not commute with the number operator N.
So eigenstates of $Q(x)$ are inclined to the 'number' axes, which scaffold the Hilbert space.



Hence, from the Vacuum state Ω there is nonvanishing transition probability to <u>any</u> localized state. This is what Malament's Theorem I is all about.

Since local observables are highly degenerate, it is convenient to work with projection operators in order to compute probabilities of finding 'eigenvalues'.

What can be measured locally is in 1:1 correspondence with the projectors in the local algebra.

Note that $\forall A(0)$, $P_{A(0)\Omega} \notin R(0)$.

so it is never a local question to ask, "are we in state $A(0)\Omega$?"

Consider $P_0 \in R(0)$.

Then measurements evaluate

$$p = \mathrm{Prob}^{\Omega}(P_0 = 1)$$

$$= \| P_0 \Omega \|^2.$$

$\forall P_1 \in R(O_1) \; \exists \; P_2 \in R(O_2)$ s.t.

$$\langle P_1 \cdot P_2 \rangle_\Omega \neq \langle P_1 \rangle_\Omega \cdot \langle P_2 \rangle_\Omega$$

Proof: For given P_1, assume $\forall P_2 \in R(O_2)$

$$\langle \Omega, P_1 P_2 \Omega \rangle = \langle \Omega, P_1 \Omega \rangle \cdot \langle \Omega, P_2 \Omega \rangle$$

Let $\hat{P}_1 = P_1 - \langle \Omega, P_1 \Omega \rangle \cdot I$

So $\langle \Omega, \hat{P}_1 P_2 \Omega \rangle = \underline{0}$

i.e. $\langle \hat{P}_1 \Omega, P_2 \Omega \rangle = \underline{0} \quad \forall P_2 \in R(O_2)$

$\Rightarrow \langle \hat{P}_1 \Omega, A(O_2) \Omega \rangle = \underline{0}$
$\forall A(O_2) \in R(O_2)$

$\Rightarrow \hat{P}_1 \Omega = \underline{0}$, since
$\{ A(O_2)\Omega : A(O_2) \in R(O_2) \}$
is dense in $\mathcal{H}$.

$\Rightarrow \hat{P}_1 = \underline{0}$, from Reeh-Schlieder theorem

$\Rightarrow P_1 = \langle \Omega, P_1 \Omega \rangle \cdot I$

$\Rightarrow P_1 = \underline{0}$ or $P_1 = I$, so if

P_1 is non-trivial (i.e. $\neq \underline{0}$ or I), Malament's 2nd theorem follows

Conclusion

Particle states in RQFT are nonlocal entities.

This is true of particle no eigenstates with definite momentum (as in collision states) or of Newton-Wigner localized particle states, which are spread everywhere in x-space (according to the Foldy-Wouthuysen transformation).

The detection of particle states in RQFT is not a local operation. Malament's localized detectors are responding to localized states of excitation of the vacuum, not to particle states.

Queries

1. What about conservation of energy?

2. Vacuum telephones?